Algebra 2

11-01A Defining and Using Sequences

Sequence

- Function whose domain are _____
- List of numbers that follow a _____
- 2, 4, 6, 8, 10
  - _____
- 2, 4, 6, 8, 10, …
  - _____

Rule

$$a_n = 2n$$

- Domain: (n)
  - Term's _____ (1^{st}, 2^{nd}, 3^{rd}…)
- Range: (a_n)
  - Term's _____ (2, 4, 6, 8…)

Write the first four terms of

$a_n = \frac{1}{2}n - 3$

$f(n) = 4^{n-1}$

Writing rules for sequences

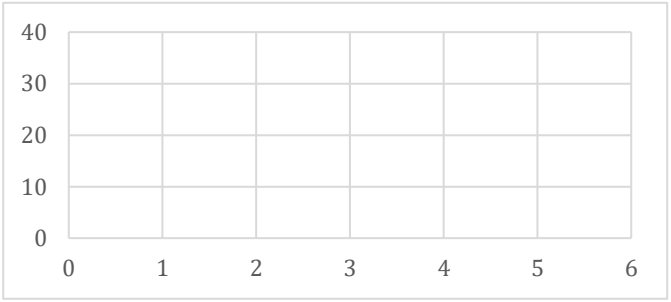
- Look for _____
- _____-and-_____
- For fractions, do top and bottom _____

$\frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \frac{2}{625}, \ldots$

3.1, 3.8, 4.5, 5.2, …

To graph

- n is like _____; a_n is like _____
- The graph will be _____
- Do _____ connect the dots!



600 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 45, 49, 59 = 15

Algebra 2

11-01B Defining and Using Series

Series

- _____ of a sequence
- 2, 4, 6, 8, … → _____
- 2 + 4 + 6 + 8 + ⋯ → _____

Summation Notation (Sigma Notation)

- Finite

$$2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i$$

_____ _____ _____

- Infinite

$$2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$$

Write as a summation

$4 + 8 + 12 + \cdots + 100$

$7 + 10 + 13 + 16 + 19$

Find the sum of the series

$\sum_{k=5}^{10} k^2 + 1$

$\sum_{i=2}^{8} \frac{2}{i}$

Some shortcut formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Find the sum of the series

$\sum_{k=1}^{10} 3k^2 + 2$

$\sum_{i=10}^{25} i$

600 #25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 47, 63, 65, 67, 69 = 15

Algebra 2

11-02 Analyzing Arithmetic Sequences and Series

Arithmetic Sequences

- Common _____ (d) between successive terms
- _____ the same number each time
- 3, 6, 9, 12, 15, …
  - d = _____

Is it arithmetic?

−10, −6, −2, 0, 2, 6, 10, …

1, −1, −3, −5, −7, …

Formula for n^{th} term

$$a_n = a_1 + (n-1)d$$

- _____

Write a rule for the n^{th} term

32, 47, 62, 77, …

51, 48, 45, 42, …

One term of an arithmetic sequence is $a_8 = 50$. The common difference is 0.25. Write the rule for the n^{th} term.

$a_{11} = 43,\ d = 5$

Two terms of an arithmetic sequence are $a_5 = 10$ and $a_{30} = 110$. Write a rule for the n^{th} term.

Sum of a finite arithmetic series

Formula

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

Consider the arithmetic series 20 + 18 + 16 + 14 + ⋯
Find the sum of the first 25 terms.

$$\sum_{i=1}^{20}(2i - 3)$$

You put money in a jar at the end of each week. The first week you put $2 in the jar, and each subsequent week you put $2 more than the previous week in the jar.

a. Write a rule for the amount of money you put in the jar at the end of the *n*th week.

b. How much money is in the jar after 9 weeks?

608 #1, 5, 9, 13, 17, 19, 21, 25, 29, 33, 37, 41, 43, 45, 50, 63, 65, 67, 72, 75 = 20

Algebra 2

11-03 Analyzing Geometric Sequences and Series

Geometric Sequence

- Created by _____ by a common _____ (r)

Are these geometric sequences?

1, 2, 6, 24, 120, …

96, 48, 24, 12, 6, …

Formula for n^{th} term

$$a_n = a_1 \cdot r^{n-1}$$

- _____

Write a rule for the n^{th} term and find a_8.

5, 2, 0.8, 0.32, …

112, 56, 28, 14, …

One term of a geometric sequence is $a_4 = 3$ and $r = 3$. Write the rule for the n^{th} term.

One term of a geometric sequence is $a_4 = -192$ and $r = 4$. Write the rule for the n^{th} term.

If two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$, write rule for the n^{th} term.

Sum of geometric series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Find the sum of the first 10 terms of
4 + 2 + 1 + ½ + ⋯

$$\sum_{i=1}^{8} 5\left(\frac{1}{3}\right)^{i-1}$$

You tell the Gospel to your friends. Four of your friends tell the Gospel to their friends, then four of each of their friends tells the Gospel, and so on. Find the total number of people who told the Gospel to others after the eighth round.

616 #1, 5, 13, 17, 19, 23, 27, 31, 35, 37, 41, 43, 44, 47, 53, 63, 65, 66, 68, 70 = 20

Algebra 2

11-04 Finding Sums of Infinite Geometric Series

Find the partial sums for n = 1, 2, 3, 4, 5 and describe what happens to S_n as n increases.

$\frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}+\frac{1}{80}+\cdots$

$4+\frac{12}{5}+\frac{36}{25}+\frac{108}{125}+\frac{324}{625}+\cdots$

Sum of an infinite geometric series

$$S=\frac{a_1}{1-r}$$

- $|r|<1$
- If $|r|>1$, then no sum (∞)

Find the sum

$\sum_{i=1}^{\infty} 2(0.1)^{i-1}$

$2+\frac{6}{4}+\frac{18}{16}+\frac{54}{64}+\cdots$

A pendulum that is released and swings freely travels 100 centimeters on the first swing. On each successive swing, the pendulum travels 96% of the distance of the previous swing. What is the total distance the pendulum travels?

Write 0.27272727… as a fraction.

Write 32.323232… as a fraction.

623 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 25, 27, 29, 31, 33, 35, 37, 39, 41 = 20

Algebra 2

11-05 Using Recursive Rules with Sequences

Explicit Rule

- Gives the n^{th} term _____
- $a_n = 2 + 4n$

Recursive Rule

- Each term is found by knowing the _____
- $a_1 = 6$; $a_n = a_{n-1} + 4$

Write the first 5 terms

$a_1 = 3,\ a_n = 2a_{n-1} - 1$

$a_1 = 2; a_n = (a_{n-1})^2 + 1$

Special Recursive Rules

- Arithmetic Sequence

  $a_n = a_{n-1} + d,\ a_1 = a_1$

- Geometric Sequence

  $a_n = r \cdot a_{n-1},\ a_1 = a_1$

Write the rules for the arithmetic sequence where $a_1 = 15$ and $d = 5$.

Explicit

Recursive

Write the rule for the geometric sequence where $a_1 = 4$ and $r = 0.2$

Explicit

Recursive

Write a recursive rule for

1, 1, 4, 10, 28, 76, …

$44,\ 11, \frac{11}{4}, \frac{11}{16}, \frac{11}{64}, \ldots$

Write a recursive rule for

$a_n = 30 - 5n$

$a_n = 12(11)^{n-1}$

Write an explicit rule for each sequence.

$a_1 = 7,\ a_n = a_{n-1} + 4$

$a_1 = -2;\ a_n = 3a_{n-1}$

A controlled laboratory contains about 500 mosquitoes. Each day, 100 new mosquitoes hatch, but the population declines 85% due to a pesticide and natural causes.

a. Write a recursive rule for the number a_n of mosquitoes at the start of the n^{th} day.

b. Find the number of mosquitoes at the start of the fourth day.

c. Describe what happens to the population of mosquitoes over time.

You borrow $2000 to travel. The loan has a 9% annual interest rate that is compounded monthly for 2 years. The monthly payment is $91.37.

a. Find the balance after the fifth payment.

b. Find the amount of the last payment.

631 #1, 5, 9, 13, 17, 21, 23, 25, 29, 33, 35, 37, 39, 41, 45, 49, 53, 59, 61, 65, 69, 70, 71, 73, 74 = 25

Algebra 2

11-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

11-02 to 11-03

Tell whether the sequence is arithmetic, geometric, or neither.

1. 4, 9, 14, 19, 24
2. 10, 20, 40, 80, 160
3. 1, 2, 6, 24, 120

Write the first four terms of the sequence.

4. $a_n = 3n + 2$
5. $a_n = 2n^2 + 1$
6. $a_1 = 3, a_n = 5(a_{n-1})$

Write the next term of the sequence, and then write the explicit rule for the *n*th term.

7. 15, 17, 19, 21, …
8. 2, 6, 18, 54, …
9. $\frac{1}{3}, \frac{3}{4}, \frac{5}{5}, \frac{7}{6}, \ldots$

Find the sum of the series. (Show work.)

10. $\sum_{i=1}^{100} 2i + 1$
11. $\sum_{i=1}^{20} 2\left(\frac{1}{3}\right)^{i-1}$
12. $\sum_{i=1}^{3} i^2$
13. $\sum_{i=2}^{5} i!$

11-04

14. $\sum_{i=1}^{\infty} 3\left(\frac{1}{2}\right)^{i-1}$

Write the repeating decimal as a fraction in lowest terms. (Show work.)

15. 0.8787878787…
16. 1.23123123123…

11-05

Write a recursive rule for the sequence.

17. 12, 19, 26, 33, 40, …
18. 10, 30, 90, 270, …
19. 3, 4, 7, 11, 18, 29, …

Word Problems.

20. (11-03) The value of a certain car is 85% of the previous year's value each year. The value of the car after the first year is $15,000. Find the explicit rule for the value of the car after *n* years. What is the value of the car after the 7th year?
21. (11-04) A company had a profit of $350,000 in its first year. Since then, the company's profit has decreased by 12% per year. If this trend continues, what is an upper limit on the total profit the company can make over the course of its lifetime?

Algebra 2 11-Review Name: _____

Answers

1. Arithmetic
2. Geometric
3. Neither
4. 5, 8, 11, 14
5. 3, 9, 19, 33
6. 3, 15, 75, 375
7. 23; $a_n = 2n + 13$
8. 162; $a_n = 2(3)^{n-1}$
9. $\frac{9}{7}$; $a_n = \frac{2n-1}{n+2}$
10. 10200
11. 3
12. 14
13. 152
14. 6
15. $\frac{29}{33}$
16. $\frac{410}{333}$
17. $a_1 = 12, a_n = a_{n-1} + 7$
18. $a_1 = 10, a_n = 3a_{n-1}$
19. $a_1 = 3, a_2 = 4, a_n = a_{n-1} + a_{n-2}$
20. $a_n = 15000(0.85)^{n-1}$; \$5657.24
21. \$2,916,666.67